PERIÓDICO TCHÊ QUÍMICA
ARTIGO ORIGINAL

ANÁLISE COMPARATIVA DE MÉTODOS NUMÉRICOS DA SOLUÇÃO DE UM PROBLEMA INVERSO UNIDIMENSIONAL DE ACÚSTICA

COMPARATIVE ANALYSIS OF NUMERICAL METHODS OF THE SOLUTION OF A ONE-DIMENSIONAL INVERSE PROBLEM OF ACOUSTICS

СРАВНИТЕЛЬНЫЙ АНАЛИЗ ЧИСЛЕННЫХ МЕТОДОВ РЕШЕНИЯ ОДНОМЕРНОЙ ОБРАТНОЙ ЗАДАЧИ АКУСТИКИ

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RESUMO

Atualmente, muitos métodos para resolver problemas inversos decorrentes de eletrodinâmica e acústica têm sido desenvolvidos, mas o desenvolvimento de sistemas práticos é necessário para combinar um grande número de equações que contribuem para a fundamentação de métodos numéricos para resolver vários problemas multidimensionais. Portanto, o principal objetivo do trabalho é uma análise comparativa de métodos numéricos para solucionar o problema acústico inverso unidimensional bem como na busca por resistência acústica. Para atingir esse objetivo, os métodos de descrição e comparação, que contribuíram para a identificação das características da impedância acústica, foram empregados. Além disso, o método da solução de diferença finita, o método de circulação do circuito diferencial e o método de iteração Landweber foram usados. Foi estabelecido que o método de inversão do esquema de diferenças é conveniente para aplicação no caso em que informações adicionais são conhecidas com precisão suficiente e a solução reconstruída é bastante suave. Foi determinado que, se uma dessas condições for violada, o método de reverter o esquema de diferenças se tornará instável. Foram investigados os problemas de correção dos problemas da equação de onda com velocidade complexa nos casos unidimensionais e espaciais. Foram obtidas fórmulas para resolver esses problemas - análogos de fórmulas clássicas. Cálculos numéricos mostraram o tipo de resultados que podem ser esperados do método considerado. O material do trabalho implica o significado prático para os professores universitários das especializações em tecnologia da informação.

Palavras-chave: *função teta de Heaviside, método de inversão de circuito de diferença, método de iteração de Landweber, analógico discreto.*

ABSTRACT

Nowadays, a large number of methods for solving inverse problems arising in electrodynamics and acoustics have been developed, but the development of practical systems is necessary to combine a large number of equations that contribute to the substantiation of numerical methods for solving various multidimensional problems. Therefore, the main goal of the work is a comparative analysis of statistical methods for solving the one-dimensional inverse acoustic problem, as well as in the search for acoustic resistance. To achieve this goal, the means of description and comparison, which contributed to the identification of the characteristics of acoustic impedance, were used. Also, the finite-difference solution method, the differential circuit circulation method, and the Landweber iteration method were used. It was established that the inversion method of the difference scheme is expedient to apply in the case when additional information is known accurately enough, and the reconstructed solution is quite smooth. It was determined that if one of these conditions is violated, the method of reversing the difference scheme becomes unstable. The problems of the correctness of the issues for the wave equation with complex velocity in the one-dimensional and spatial cases were investigated. Formulas for solving these problems were obtained – analogs of classical formulas. Numerical computations show the kind of results that may be expected from the method under consideration. The materials of the paper imply the practical significance for the university teachers of the information technology specializations.

Keywords: *Hevisayd's theta-function, final and differential look, method of the circulation of the differential scheme, method of iterations of Landveber, discrete analog.*

АННОТАЦИЯ

К настоящему времени существуют различные методы решения обратных задач, возникающих в электродинамике, акустике, но разработка практических систем необходима для объединение большого числа уравнений, которые способствуют обоснованию численных методов решения многомерных различных задач. Поэтому основная цель работы заключается в сравнительном анализе численных методов решения одномерной обратной задачи акустики, а также в поиске акустического сопротивления. Для достижения поставленной цели авторами были использованы методы описания, сравнения, которые поспособствовали выявлению особенностей акустического импеданса. Также в работе применены конечно-разностный метод, метод циркуляции дифференциальной схемы и метод итераций Ландвебера. Установлено, что метод обращения разностной схемы целесообразно применять в случае, когда дополнительная информация известна достаточно точно и решение не требует дополнительных подходов. Определено, что при нарушении одного из этих условий метод обращения разностной схемы становится неустойчивым. Исследованы вопросы корректности задач для волнового уравнения с комплексной скоростью в одномерном и пространственном случаях. Получены формулы решения этих задач – аналоги классических формул. Численные расчеты показывают результаты, которые возможно ожидать от использованного метода. Материалы статьи предполагают практическую значимость для преподавателей вузов, которые специализируются на информационных технологий.

Ключевые слова: *тэта-функция Хевисайда, метод обращения разностной схемы, метод итераций Ландвебера, дискретный аналог.*

1. INTRODUCTION

As is known, most of the inverse problems of geophysics is devoted to determining the speed of sound in the framework of the wave equation or its various approximations. It is assumed that other parameters of the medium, such as the density and absorption of the medium, are constant and known (Evstigneev *et al.*, 2016; Chkadua, 2017; Aliev and Isayeva, 2018; You *et al.*, 2018). Knowledge of this single parameter of the medium – the speed of sound is sufficient in many practical cases. At the same time, it is evident that in many essential inverse problems it is impossible to confine oneself to this approximation and it is necessary to use more accurate models of the structure of the medium and introduce other additional parameters that allow a more adequate description of reality (Gibson, 2018; Safarov, 2018; Pejić *et al.*, 2018). These multi-parameter models provide more complete information about the structure of the medium and are therefore of great interest. Naturally, the processes occurring in such media are described by more complex equations than the wave equation.

The creation and justification of numerical methods for solving inverse and incorrect problems is an urgent problem, firstly, due to the practical importance of reverse and incorrect problems, and secondly, due to the need to create effective algorithms for solving multidimensional inverse and incorrect acoustics issues and electrodynamics.

It is known that the optimization method is an effective method for solving inverse problems. This method consists in minimizing a specific function concerning the required parameters. The required parameters are the values of the determined quantities at given points in space. Various iterative procedures are used as minimization methods. The quality of one or another numerical minimization method is characterized by efficiency, i.e., the number of operations necessary to obtain a given accuracy of the solution, and the ultimate accuracy with which you can approach the minimum point. The latter determines the ultimate accuracy of solving the inverse problem. Not always, but as a rule, a drop in the efficiency of the algorithm is accompanied by a decline in the accuracy of the solution to the problem.

The one-dimensional return problem of acoustics is considered in “Iterative methods of the solution of the return and incorrect tasks with data on the part of border” (Kabanikhin *et al.*, 2006).

To increase the efficiency of acoustic resistance, a multilevel adaptive algorithm is used. The algorithm is based on dividing the entire algorithm for solving the inverse problem into a series of consecutive levels. When moving from one level to another, the number of parameters changes adaptively depending on the size of the functional and the rate of convergence – a multi-level adaptive method (Pleshchinskii *et al.*, 2018; Gamzaev, 2019; Leinartas and Shishkina, 2019). This method can only be used to obtain an approximate (trend) solution of the inverse problem since when moving from level to level; the answer is interpolated over several points, i.e., essentially averaging over these points is performed (Equations 1–4), where it is necessary $\sigma(x) > 0,\ x > 0,\ \sigma \in C^1[0,\infty)$. It is required to find the solution of a direct task (Equations 1–3) $u(x,t)$ and acoustic rigidity of the environment $\sigma(x)$ according to additional information (Equation 4). It is known (Romanov, 1973), that the solution of the direct problem (Equations 1–3) has the form (Equation 5), Where $\tilde{u}(x,t)$ – continuity $x \geq 0$ and is sufficiently smooth for $t > x > 0$ function, Equation 6 – Heaviside theta function.

This equation is a kind of competitor concerning the equation with the equivalent inverse problem when describing waves in a rectangular channel. Substituting (Equation 5) in (Equations 1–4), the equivalent inverse problem relatively $u(x,t)$ and $s(x)$ is obtained.

The class of nonlinear evolution equations has several additional properties whose origin is related to the main property of this class of nonlinear evolution equations, that is, although temporal evolution is nonlinear in the configuration space, it is connected (by spectral transformation) with simple linear development in spectral space (Equations 7–10).

Thus, this paper aimed to study and substantiate numerical methods for solving a one-dimensional inverse problem of acoustics, since they are important both from a theoretical and a practical point of view.

2. MATERIALS AND METHODS

A difference scheme is a finite system of algebraic equations containing a differential equation and additional conditions (for example, boundary conditions and / or initial distribution). Thus, difference schemes are used to reduce a differential problem of a continuous nature to a finite system of equations, the numerical solution of which is fundamentally possible on computers. Algebraic equations associated with the differential equation are obtained using the difference method, which distinguishes the theory of difference schemes from other numerical methods for solving differential problems.

The solution of the difference scheme is called the approximate solution of the differential problem. The authors introduce the net $x = ih, t = kh.$ represented by the equation (6) in finite difference form (3) (Equation 11) from where having expressed u_{i+1}^{k} (Equation 12) will be obtained. The authors approximate a boundary condition (7) (Alontseva and Gilev, 2011; Samarskiy, 1971) (Equation 13).

Analytical methods are mainly used to study single-phase or two-phase linear models of various physical processes. It is almost impossible to obtain an exact solution to a nonlinear model using existing analytical methods. In connection with this, numerical methods for solving such problems are increasingly used in modeling, but with their help, it is not always possible to accurately track the law of motion of the phase boundary.

Thus, assuming that all considered functions rather smooth, the authors will write down the return task (Equations 7–10) in a final and differential look (Equations 14–17).

Conclusion of discrete analog of a formula of Dalamber. The d'Alembert formula can found a solution to the initial – boundary value problem with a homogeneous first-order boundary condition if the primary functions continue along the entire axis odd relative to the origin. A representation of the d'Alembert type here not only provides information on the structure of the general solution but can also be used as the basis for creating a computational scheme for solving the initial-boundary-value problem. Using a known technique (Denisov, 1994; Kabanikhin, 1984; Kabanikhin *et al*., 2004), discrete analog of a formula of Dalamber will be received. Shifting indexes in (Equation 14) a chain of equalities (Equations 18-23) will be received. From where (Equation 24) will be obtained. From where Dalamber's formula in a discrete look (Equations 25–30) will be received or, having made replacement s = i – m, (Equation 31) will be obtained.

3. RESULTS AND DISCUSSION:

3.1. Method of the circulation of the differential scheme

The method of circulating the differential scheme is quite natural from the physical point of view, since it uses the theory of characteristics along which, as a rule, the basic information about the features of solving the direct problem and the medium under investigation is distributed. In the computational aspect (by the number of operations), this method is equivalent to a single solution to the corresponding direct problem and allows parallelization of the calculation procedure (Xu *et al.*, 2016; Guliyev and Nasibzadeh, 2018). Substituting (Equation 31) expression in (Equation 13) and considering (Equation 15), a formula of calculation of unknown function will be received (Kornilov, 2005; Kraht, 2005; Nurseitova and Tyulepberdinova, 2008) (Equations 32–33). The authors will carry out calculations from the border $i = 0$, along with characteristics, as shown in the scheme of Figure 1–15. At first, we will be counted s_0, then, knowing the value u_0^2, s_1 will be calculated. Further from u_0^4, calculating along the characteristic, we define s_2 and so on (Appendix 1).

For check of the work of algorithm on the solution of the returned task, the authors will set exact function $s(x)$ **;** then, a direct problem will be solved; a decision trail $x = 0$ will be picked up, thereby function $g(t)$ additional information will be defined. The authors will describe the scheme of the solution of a direct task (Equations 34–38). Here on the known s_i, calculating along with characteristics, we define g_i (Figure 2).

3.2. Method of iterations of Landveber

For check of work of algorithm on the solution of the returned task the authors will set exact function s(x), then a direct problem will be solved by method of the circulation of the differential scheme, a decision trail at x = 0, will be picked up, thereby function g(t) – additional information will be defined**.** Now we will describe computing experiments for various types of functions s(x) (Nurseitova *et al.*, 2010; Samarskiy and Vabishevich, 2004). *Linear function s(x), noise parameter* $\varepsilon \approx 0.01$

For the following parameters N = 200, l= 1, h = l/N = 0.005 with function of a look s(x) = – 0.5x – 1 the direct task was solved and function is received g(t). After the addition of a random error function g(t), I assumed an air results of computing experiment are presented in Figure 3. The authors will carry out the comparative analysis of methods of the solution of the return problem of acoustics (Table 1). Results of numerical calculations for function are shown (Equation 39) with parameters $N = 200, l = 1, h = l / N = 0.005$ and indignation of entrance data $\|\overline{q}_3 - q_3\| = 0.0142$.

The technology for solving the inverse problem for objects with dispersion of sound speed allows us to formulate a more advanced approach to the creation of ultrasonic non-destructive testing techniques. Here you select a frequency range that captures the region with the dispersion of the speed of sound.

4. CONCLUSIONS:

The approximate decision received a method of the circulation of the differential scheme meets to exact better than an approximation of a way of iteration of Landveber, if data exact. Apparently, from Figure 4.4 at inexact data, the method of the circulation of the differential scheme disperses as it is unstable. In Figures 4–13 it is visible that the approximate decision received approximation of a method of iteration of Landveber well meets to exact and at noisy data, though convergence speed the low. Receiving algorithm for discrete analog of a method of iteration of Landveber is followed by more bulky calculations, than approximation of a method of iteration of Landveber, but apparently from the table, discrete analog of a method of iteration of Landveber gives the best approach for twice smaller number of iterations, than approximation of a method of iteration of Landveber. Thus labor input of realization of one iteration approximation of a way of repetition of Landveber and discrete analog of a method of iteration of Landveber have one order, that is this machine time spent for the performance of one iteration has one order.

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$$u_{tt} = u_{xx} - 2\frac{\sigma'(x)}{\sigma(x)}u_x, \quad x > 0, t > 0, \tag{Eq. 1}$$

$$u\,|_{t<0} \equiv 0, \quad x > 0, \tag{Eq. 2}$$

$$u_x\,|_{x=+0} = \gamma\delta(t), \quad t > 0, \tag{Eq. 3}$$

$$u\,|_{x=+0} = g(t), \quad t > 0. \tag{Eq. 4}$$

$$u(x,t) = s(x)\theta(t-x) + \tilde{u}(x,t), \tag{Eq. 5}$$

$$s(x) = -\gamma\sqrt{\sigma(x)/\sigma(+0)}, \quad \theta\ . \tag{Eq. 6}$$

$$u_{tt} = u_{xx} - 2\frac{s'(x)}{s(x)}u_x, \quad t > x > 0 \tag{Eq. 7}$$

$$u_x\,|_{x=0} = 0, \quad t > 0, \tag{Eq. 8}$$

$$u(x, x+0) = s(x), \quad x > 0, \tag{Eq. 9}$$

$$u\,|_{x=+0} = g(t), \quad t > 0. \tag{Eq. 10}$$

$$\frac{(u_i^{k+1} - 2u_i^k + u_i^{k-1}}{h^2} = \frac{(u_{i+1}^k - 2u_i^k + u_{i-1}^k}{h^2} - 2\frac{(s_{i+1} - s_{i-1})}{h(s_{i+1} - s_{i-1})} \cdot \frac{s_{i+1}^k - s_{i-1}^k}{2h}, \tag{Eq. 11}$$

$$u_{i+1}^k = \frac{(u_i^{k+1} + u_i^{k-1})(s_{i+1} + s_{i-1}) - 2u_{i-1}^k s_{i+1}}{2s_{i-1}}. \tag{Eq. 12}$$

$$u_1^k = u_0^k + h\left.\frac{\partial u}{\partial x}\right|_{x=0} + \frac{h^2}{2}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x=0} + 0(h^2) = u_0^k + \frac{h^2}{2}\left(\frac{u_0^{k+1} - 2u_0^k + u_0^{k-1}}{h^2} + 2\frac{s'(0)}{s(0)}\left.\frac{\partial u}{\partial x}\right|_{x=0}\right) + 0(h^2)$$
$$= \frac{u_0^{k+1} + u_0^{k-1}}{2} + 0(h^2). \tag{Eq. 13}$$

$$u_{i+1}^k = \frac{(u_i^{k+1} + u_i^{k-1})(s_{i+1} + s_{i-1}) - 2u_{i-1}^k s_{i+1}}{2s_{i-1}}, \tag{Eq. 14}$$

$$u_1^k = \frac{u_0^{k+1} + u_0^{k-1}}{2}, \tag{Eq. 15}$$

$$u_i^i = s_i, \tag{Eq. 16}$$

$$u_0^k = g_k. \tag{Eq. 17}$$

$$u_i^{k+1} = u_{i-1}^{k+2} + u_{i-1}^k - u_{i-2}^{k+1} + h^2 Q_{i-1}^{k+1},$$

$$u_i^{k+1} = u_{i-1}^{k+2} + u_{i-1}^{k} - u_{i-2}^{k+1} + h^2 Q_{i-1}^{k+1}, \quad \text{(Eq. 18)}$$

$$u_{i-1}^{k+2} = u_{i-2}^{k+3} + u_{i-2}^{k+1} - u_{i-3}^{k+1} + h^2 Q_{i-2}^{k+2}, \quad \text{(Eq. 19)}$$

$$u_{i-j+1}^{k+j} = u_{i-j}^{k+j+1} + u_{i-j}^{k+j-1} - u_{i-j-1}^{k+j} + h^2 Q_{i-j}^{k+j}, \quad \text{(Eq. 20)}$$

$$\vdots \quad j = i-2 \quad \text{(Eq. 21)}$$

$$u_3^{k+i-2} = u_2^{k+i-1} + u_2^{k+i-3} - u_1^{k+i-2} + h^2 Q_2^{k+i-2}, \quad \text{(Eq. 22)}$$

$$u_2^{k+i-1} = u_1^{k+i} + u_1^{k+i-2} - u_0^{k+i-1} + h^2 Q_1^{k+i-1}, \quad \text{(Eq. 23)}$$

$$u_{i+1}^{k} = u_i^{k-1} + u_1^{k+i} - u_0^{k+i-1} + h^2 \sum_{j=1}^{i} Q_j^{k+i-j}. \quad \text{(Eq. 24)}$$

$$u_i^{k-1} = u_{i-1}^{k-2} + \frac{g^{k+i-1} - g^{k+i-3}}{2} + h^2 \sum_{j=1}^{i-1} Q_j^{k+i-j-2}, \quad \text{(Eq. 25)}$$

$$u_i^{k-1} = u_{i-1}^{k-2} + \frac{g^{k+i-1} - g^{k+i-3}}{2} + h^2 \sum_{j=1}^{i-1} Q_j^{k+i-j-2}, \quad \text{(Eq. 26)}$$

$$u_{i-m+1}^{k-m} = u_{i-m}^{k-m-1} + \frac{g^{k+i-2m+1} - g^{k+i-2m-1}}{2} + h^2 \sum_{j=1}^{i-m} Q_j^{k+i-j-2m}, \quad \text{(Eq. 27)}$$

$$\vdots \quad m = i-1 \quad \text{(Eq. 28)}$$

$$u_2^{k-i+1} = u_1^{k-i} + \frac{g^{k-i+3} - g^{k-i+1}}{2} + h^2 \sum_{j=1}^{1} Q_j^{k-i-j+2}, \quad \text{(Eq. 29)}$$

$$u_1^{k-i} = u_0^{k-i-1} + \frac{g^{k-i+1} - g^{k-i-1}}{2}. \quad \text{(Eq. 30)}$$

$$k = i+1. \quad \text{(Eq. 31)}$$

$$u_{i+1}^{k} = \frac{g^{k+i+1} - g^{k-i-1}}{2} + h^2 \sum_{m=0}^{i-1} \sum_{j=1}^{i-m} Q_j^{k+i-j-2m}, \quad \text{(Eq. 32)}$$

$$u_{i+1}^{k} = \frac{g^{k+i+1} - g^{k-i-1}}{2} + h^2 \sum_{s=1}^{i} \sum_{j=1}^{s} Q_j^{k-i-j+2s}. \quad \text{(Eq. 33)}$$

$$s_{i+1} = s_{i-1} \frac{u_i^{i+2} + s_i}{2s_{i-1} - s_i - u_i^{i+2} + 2u_{i-1}^{i+1}} \quad \text{(Eq. 34)}$$

$$u_{i+1}^{k} = \frac{2u_{i+1}^{k} s_{i-1} + 2u_{i-1}^{k} s_{i+1}}{s_{i+1} + s_{i-1}} - u_i^{k-1}, \quad \text{(Eq. 35)}$$

$$u_0^{k+1} = 2u_1^k - u_0^{k-1}, \quad \text{(Eq. 36)}$$

$$u_i^i = s_i, \quad \text{(Eq. 37)}$$

$$u_0^k = g_k. \quad \text{(Eq. 38)}$$

$$s(x) = -1/2x - 1, \quad \text{(Eq. 39)}$$

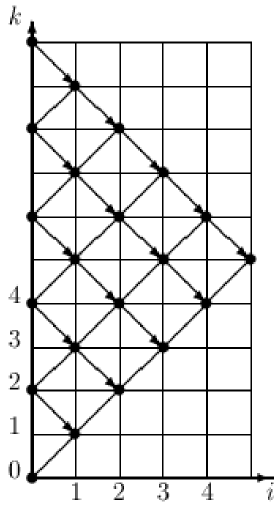


***Figure 1**. Return task*

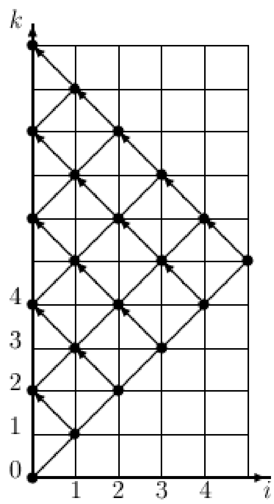


***Figure 2**. Direct task*

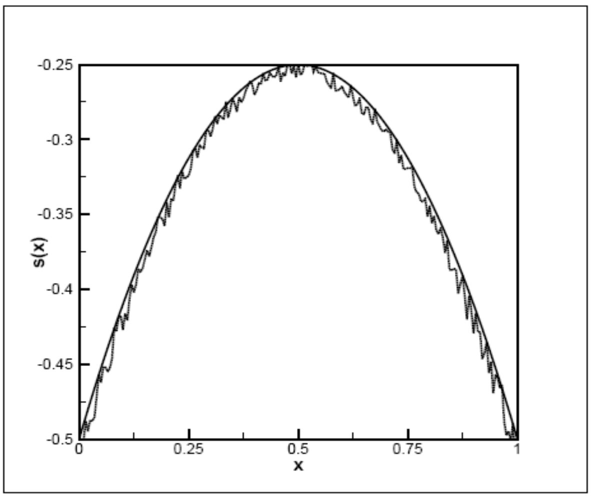


***Figure 3**. Schedules of restoration parabolic functions s(x) at indignation* $\|\tilde{s} - s\| \cong 0.0019$. **The continuous line – the exact decision s(x); the dot line – the approximate decision*

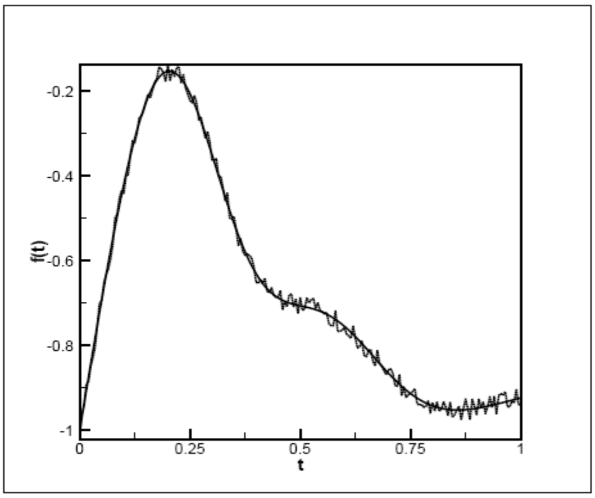


***Figure 4**. Function graph of additional information g(t) for periodic type of the decision at* $\|\tilde{g} - g\| \cong 0.014$ *. *The continuous line – the exact decision s(x); the dot line – the approximate decision*

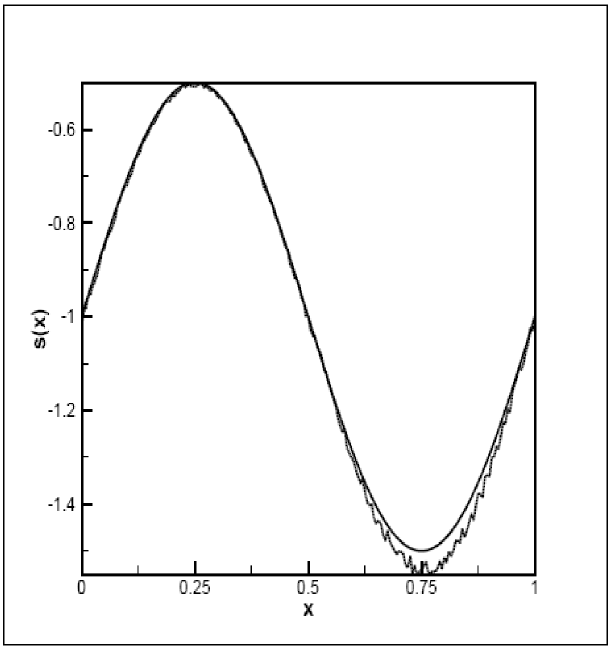


***Figure 5**. Restoration of periodic function s (x) at* $\|\tilde{s} - s\| \cong 0.023$*. *The continuous line – the exact decision s(x); the continuous line – the exact decision*

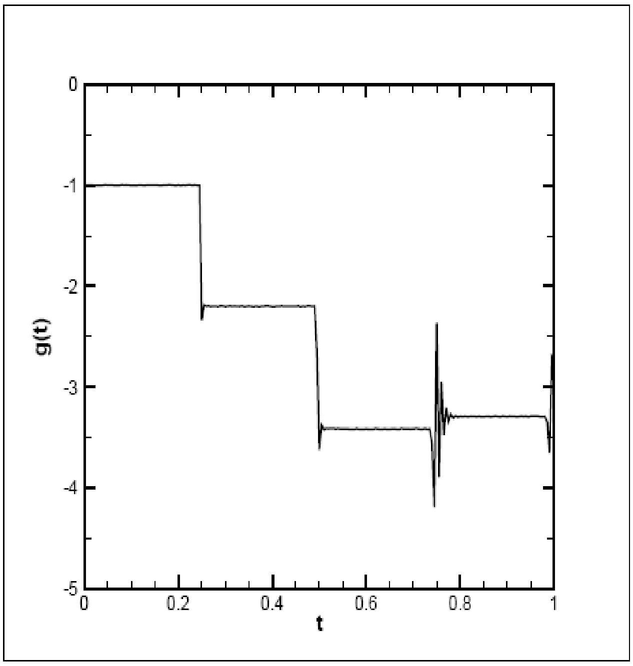


Figure 6. *Function graph of additional information g(t) for step type of the decision at* $\|\tilde{g} - g\| \cong 0.003$. **The continuous line – exact function g(t); the dot line – the indignant function of additional information*

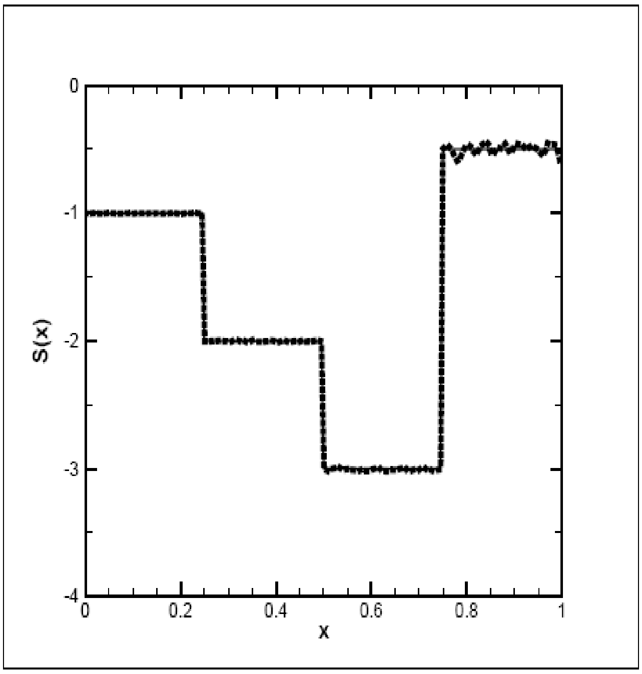


Figure 7. *Function restoration s(x) by method a method of the circulation of the differential scheme at* $\|\tilde{s} - s\| \cong 0.02$. **The continuous line – exact function s(x); dot line approximate decision*

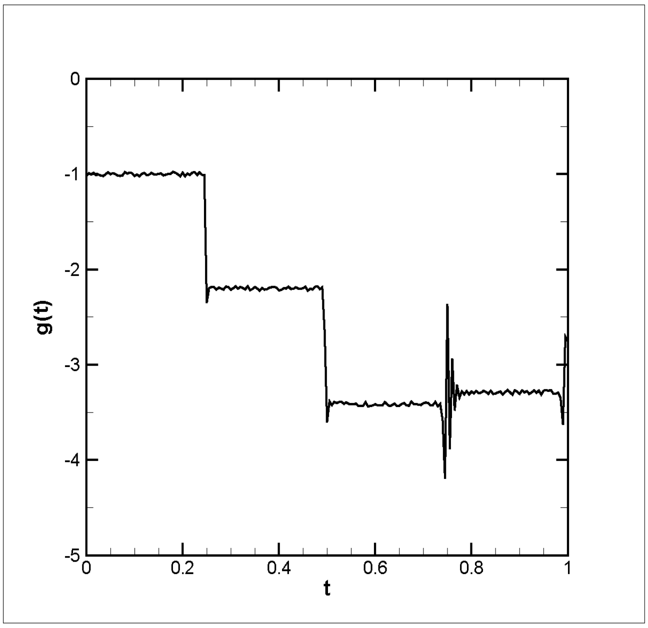


Figure 8. *Function graph of additional information g(t) for step type of the decision at* $\|\tilde{g} - g\| \cong 0.01$.
**The continuous line – exact function g(t); dot line indignant function of additional information*

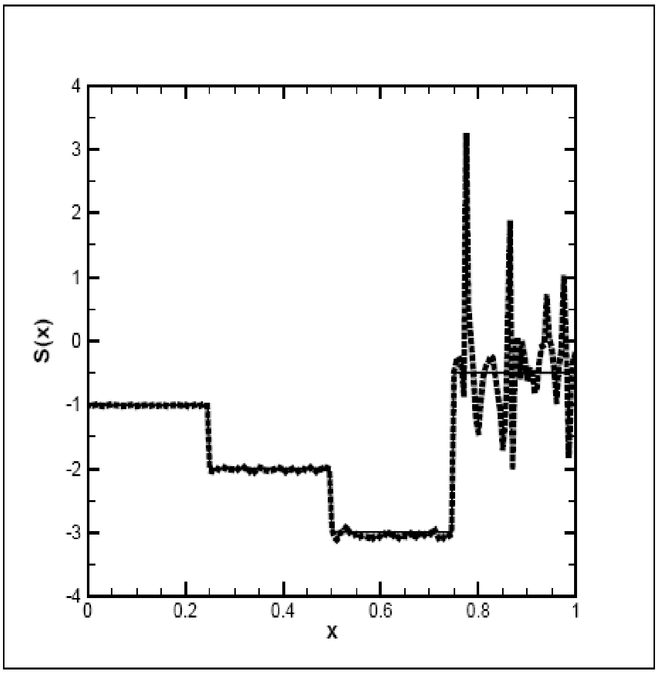


Figure 9. *Function restoration s(x) by method a method of the circulation of the differential scheme at* $\|\tilde{s} - s\| \cong 0.02$. **The continuous line – exact function s(x); dot line approximate decision, the circulation of the differential scheme restored by method a method*

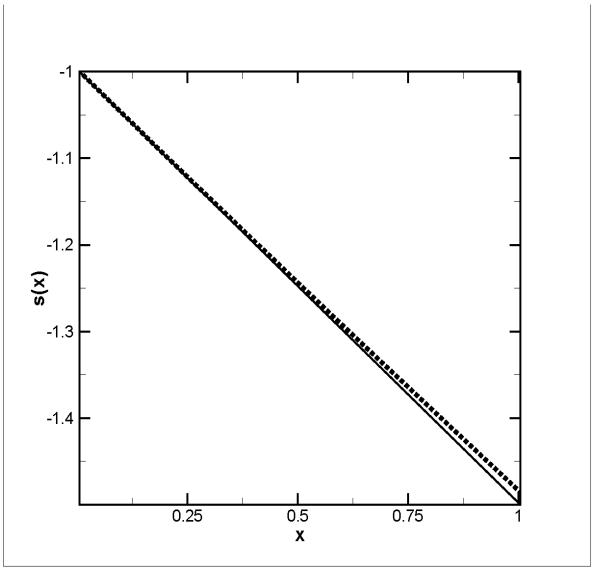


***Figure 10**. Schedule of restoration of periodic function s (x) by method approximation of a method of iteration of Landveber. ***The continuous line – exact function s(x); dashed line – the approximate decision, restored by method approximation of a method of iteration of Landveber*

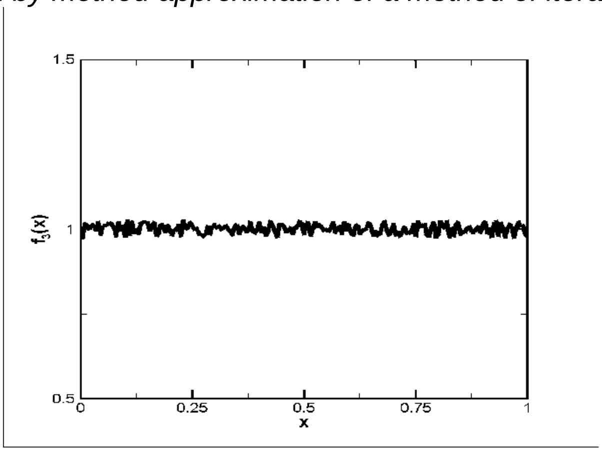


***Figure 11**. Schedule of noisy function* $f_3(x)$

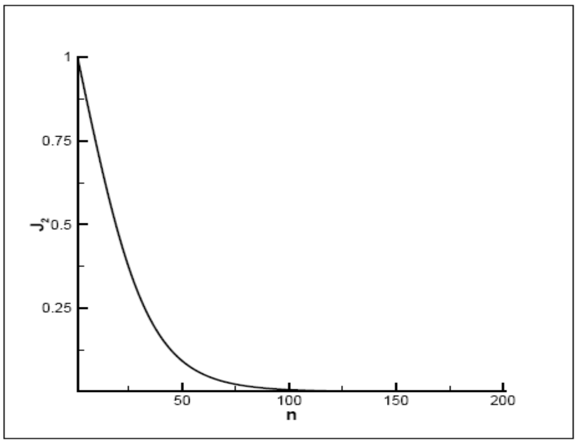


***Figure 12**. Schedule of functionality $J_2(q^n)$*

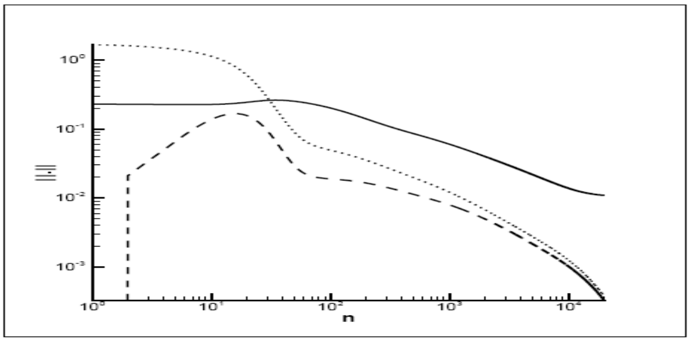
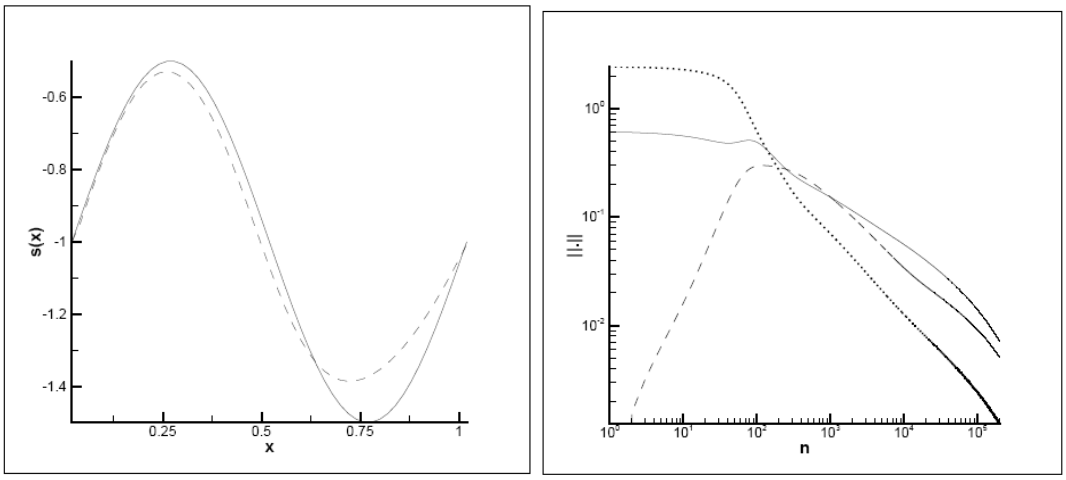


Figure 13*. Schedule of functionality of a method approximation of a method of iteration of a method.*
**The continuous line – values of functionality* $J_1(q^n)$; *dashed line – values of functionality* $J_2(q^n)$; *the dot line – values of functionality* $J_3(q^n)$;



a) *b)*

Figure 14*. Schedule of restoration of periodic function s(x). *The continuous line – exact function s(x); dashed line – the approximate decision restored by method approximation of a method of iteration of Landveber*

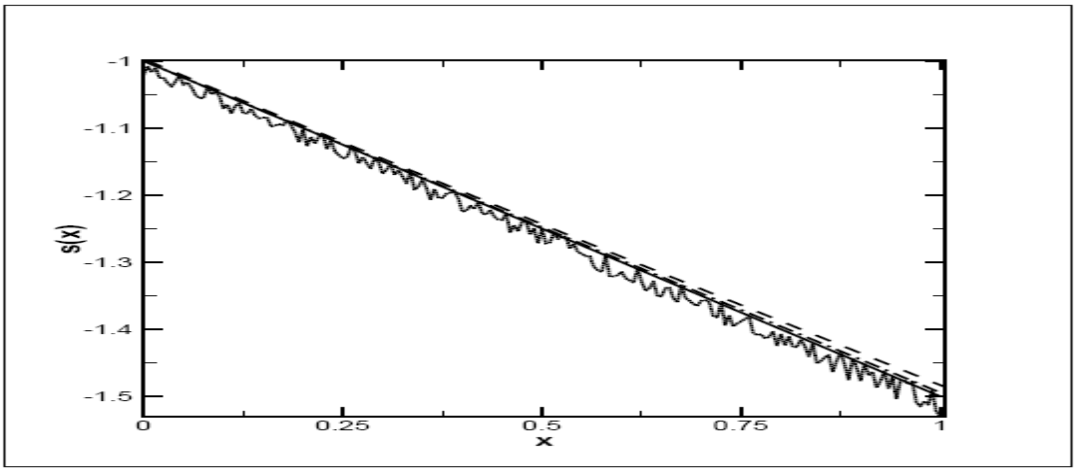


Figure 15*. Comparison of various methods. *The continuous line – exact function s(x); the dot line – the approximate decision restored by method method circulation of the differential scheme; dashed line – the approximate decision restored by method approximation of a method of iteration of Landveber; the dash-dotted line – the approximate decision restored by method discrete analog of a method of iteration of Landveber*

Table 1. Comparative table of methods

Method / Criterion	Method circulation of the differential scheme	Approximation of a method of iteration of Landveber	Discrete analog of a method of iteration of Landveber
$\|s - s_{ex}\|$	$1.56 \cdot 10^{-2}$	$9.88 \cdot 10^{-3}$	$2.90 \cdot 10^{-3}$
Scheme	the obvious	the iterative	the iterative
Iterations	-	198	93

Appendix 1

Algorithm of the solution of a Direct task:

1. Value on a Equation 17 is calculated
$s_0 = u_0^0 = g_0$.
2. s_1 is calculated:
(a) on a Equation 17 value $u_0^2 = g_2$;
(b) calculating on a Equation 15 value $s_1 = u_1^1 = \dfrac{u_0^2 + u_0^0}{2}$.
3. s_2 is calculated:
(a) on a Equation 17 value $u_0^4 = g_4$;
(b) calculating on a Equation 15 value $u_1^3 = \dfrac{u_0^4 + u_0^2}{2}$;
(c) calculating on a Equation 21 value s_2.
4. s_3 is calculated:
(a) on a Equation 17 value $u_0^6 = g_6$;
(b) calculating on a Equation 15 value $u_1^5 = \dfrac{u_0^6 + u_0^4}{2}$;
(c) calculating on a Equation 14 value u_2^4;
(d) calculating on a Equation 21 value s_3.
5. And so on, $s_i, i = \overline{4, N}$ is calculated:
(a) on a Equation 17 value $u_0^{2i} = g_{2i}$;
(b) calculating on a Equation 15 value $u_1^{2i-1} = \dfrac{u_0^{2i} + u_0^{2i-2}}{2}$;
(c) calculating on a Equation 14 along characteristics and value of functions $u_2^{2i-2}, \ldots, u_{i-1}^{i+1}$;
(d) calculating on a Equation 21 value s_i.